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Laminar thermal convection in a vertical slot: transient behaviour and thermal diffusivity determination

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Abstract—A quasi-static theory is derived for the relaxation of the velocity field resulting from a temperature change in an infinite vertical fluid layer when the temperature gradient is perpendicular to the gravity vector. Taking the viscosity temperature dependence into account, experimental results fit remarkably well the reference data for the thermal diffusivity of the studied systems. The laser–Doppler velocimetry (LDV) technique, applied to transient velocity measurements, is thus recommended to determine the thermal diffusivity in fluids.

INTRODUCTION

In a previous paper [1], the present authors have already pointed out one of their objectives in thermal convection in a vertical slot: a quantitative analysis of the conductive regime in the presence of a horizontal temperature gradient. Therefore they considered an infinite vertical slot (at least in the theoretical part of their work), approximated in the experimental part by a slot of aspect ratio, height to width, equal to 28. In such conditions, it is hoped that the horizontal heat transfer, due to the horizontal velocity component, will remain negligible, or in other words, that the Nusselt number will remain equal to unity (this is the so-called "conduction regime"). In such a situation the theoretical analysis appears to be a student exercise, since one has to integrate linear differential equations in order to determine the velocity field. It has been known for a long time that in a slot of finite aspect ratio there is a transition to a boundary layer type convection regime [2–4] at high temperature gradients (or at high Grashof numbers), followed by a transition to a multicellular and next a time dependent regime.

In [1], it was demonstrated that, when the density is a linear function of the temperature, the conduction regime prevails when the Grashof number remains smaller than 2000 in a slot of aspect ratio 28. Since the velocity amplitudes have to remain small for the validity of the conduction regime, this should be the case for water near its density maximum occurring at 4°C, and for highly viscous fluids like glycerine. The experimental results reported in [1], concerning the vertical velocity profiles obtained by laser–Doppler velocimetry, fit remarkably well the theoretical predictions, even when non-Boussinesq effects are included.

In the present paper, still devoted to the conduction regime, we study the transient behaviour due to a sudden change in boundary conditions. The initial state is an equilibrium state at uniform initial temperature T_i. At time $t = 0$ the boundary conditions are changed and the new temperature is T_f. The time evolution of the temperature field was studied a long time ago by Fourier, and the transient behaviour is solely governed by the thermal diffusivity κ of the system: the temperature field relaxes more or less rapidly to the constant T_f value depending on κ. But at any small time, there is a temperature gradient, implying in fluids a density gradient and therefore thermal convection. The way the velocity field relaxes to the equilibrium rest state can be easily derived, at least in the quasi static approximation.

Despite the academic nature of the problem, the relaxation of the velocity field to the rest state is linked to the relaxation of the temperature field to its constant T_f value, i.e. to the thermal diffusivity.

Therefore we show in this paper that the thermal diffusivity of liquids can be obtained by recording the velocity, at a given point, e.g. by laser–Doppler velocimetry, provided that correct equations are used for the time behaviour of the velocity field. The main interest is the duration of an experiment (typically 1 min) together with its non invasive nature.

Thus, in contradistinction with usual techniques, we measure the velocity field, instead of trying to completely eliminate the effects of natural convection, like in the hot wire technique applied by J. J. De Groot *et al.* [5] to gases and the method developed by Van der Held *et al.* [6] applied to liquids. There is of course no problem with convection in the case of solids (see e.g. the flash method described by Parker *et al.* [7]).

BASIC EQUATIONS

The thermal diffusivity κ defines the time constant in the energy equation for time dependent processes:

NOMENCLATURE

A_n	Fourier coefficient
B_n	Fourier coefficient
A, B, C	numerical coefficients
g	gravity
h	length scale
n	integer
p	pressure
Pr	Prandtl number, ν/κ
t	time
T	temperature
T_f	final temperature
T_i	initial temperature
T_n	particular solution of the heat equation
V_c	characteristic velocity
V_x, V_y, V_z	velocity components
x	horizontal coordinate
z	vertical coordinate.

Greek symbols

α	thermal expansion coefficient
δ	viscosity coefficient
θ	reduced temperature
κ	thermal diffusivity
λ_n	wave number
μ	dynamic viscosity
μ_f	dynamic viscosity at temperature T_f
ν	kinematic viscosity
ρ	density
ρ_f	density at temperature T_f
τ	thermal relaxation time
τ_0, τ_1, τ_2	successive approximations of τ.

$$\partial T/\partial t = \kappa \nabla^2 T. \qquad (1)$$

Let us consider a one-dimensional problem $x \in [-h; +h]$.

A particular solution of equation (1) is:

$$T_n = [A_n \sin(\lambda_n x) + B_n \cos(\lambda_n x)]\, e^{-\sigma_n t} \qquad (2)$$

with $\sigma_n = \kappa\lambda_n^2$. The general solution, $T = \Sigma T_n$, has to satisfy the initial, final and boundary conditions:

$T = T_i$(initial temperature) for $t < 0$

$T = T_f$(final temperature) for $t \geqslant 0$ at $x = \pm h$

$T = T_f$(final temperature) everywhere for $t \to \infty$.

One gets

$$\theta = 1 - 4/\pi \sum_{n=0}^{\infty} (-1)^n/(2n+1) \cos[(2n+1)\pi x/2h]\, e^{-\sigma t} \qquad (3a)$$

where

$$\theta = (T - T_i)/(T_f - T_i) \qquad (3b)$$

$$\sigma = \kappa(2n+1)^2\pi^2/4h^2. \qquad (3c)$$

Let us now write the equations governing thermal convection in an infinite vertical slot of width $2h$, the infinite vertical walls being located at $x = +h$ and $x = -h$. The horizontal axis is thus the x axis, and z will be the vertical axis. Since the slot is supposed infinite on the z direction one has $V_x = 0$, and from the mass balance equation, one gets $\partial V_z/\partial z = 0$ and therefore V_z is x and time dependent only. Thus in the so-called conductive regime the relevant equations read:

$$\partial p/\partial x = 0 \qquad (4)$$

$$\rho_f \partial V_z/\partial t = \mu \partial^2 V_z/\partial x^2 - \partial p/\partial z - g\rho(T(t)) \qquad (5)$$

with p the pressure, μ the dynamic viscosity, V_i the velocity components, g the acceleration of the gravity, and ρ the density given by the equation of state

$$\rho = \rho_f[1 - \alpha(T - T_f)] \qquad (6)$$

with α the thermal expansion coefficient, T_f the reference final temperature, and ρ_f the reference density at temperature T_f.

Because the temperature is time dependent, the density will also be time dependent. In consequence, the velocity field generated by the temperature gradient will also be time dependent. Let us now *suppose* that the slow process is the relaxation of the temperature field, thus that the velocity field "follows" immediately the temperature field, or in other words that κ is small (compared to the kinematic viscosity ν). In this "quasi-static" approximation, the left hand side of equation (5) can be neglected, and then its space integration becomes straightforward.

The assumption of a quasi-static process simply implies the condition

$$Pr \gg 1 \qquad (7)$$

where Pr is the Prandtl number $Pr = \nu/\kappa$, and which is probably true for most organic liquids and water. The assumption fails of course for liquid metals of high thermal diffusivity, for which the relaxation of the thermal field is the fast process.

Putting thus the left-hand side (LHS) of equation (5) equal to zero, eliminating the pressure by cross-differentiation between equations (4) and (5), the governing differential equation for V_z simply reads (supposing a constant viscosity)

$$\mu \frac{\partial^3 V_z}{\partial x^3} = -\alpha g \rho_f \frac{\partial T}{\partial x} \qquad (8)$$

submitted to the usual no-slip boundary conditions

$$V_z(-h, t) = V_z(+h, t) = 0 \qquad (9)$$

together with the zero flux condition across any horizontal surface.

$$\int_{-h}^{+h} V_z(x,t)\,\mathrm{d}x = 0. \tag{10}$$

Integrating three times equation (8) with respect to x, using equations (3) to calculate the temperature gradient $\partial T/\partial x$, and applying the conditions (9) and (10) to determine the three integration constants, one finds for the time dependent vertical velocity profile, using the dimensionless co-ordinate $X = x/h$

$$\frac{V_z(x,t)}{V_c} = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-(2n+1)^2(t/\tau)}}{(2n+1)^3} \times \left[(-1)^n \cos\frac{(2n+1)\pi X}{2} + \frac{3}{\pi}\frac{1}{(2n+1)}(X^2-1)\right] \tag{11}$$

where τ is the thermal relaxation time

$$\tau = \frac{4h^2}{\pi^2 \kappa} \tag{12}$$

and V_c a characteristic velocity

$$V_c = \frac{16}{\pi^3}\frac{g\alpha|T_f - T_i|h^2}{\nu}. \tag{13}$$

In Fig. 1, the velocity profile is tabulated for "large" times ($t = 0.5\,\tau$; τ; 2τ and $5\,\tau$), retaining in the summation as many terms as needed to achieve an unnecessary six digits precision ("unnecessary", as far as a comparison with experiments is concerned). The "small" time behaviour ($t < 0.1\,\tau$) needs of course more terms, and will not be displayed since it will not be used further. As a matter of fact, at time $t = 0$, $V_z = 0$ everywhere. When $t > 5\,\tau$, the velocity field becomes non measurable. The time relaxation of the velocity field to the rest state is thus related to the thermal diffusivity. In order to determine from this relaxation the value of κ, there is no need to record the whole velocity field. It is sufficient to dispose of time measurements of the velocity at a given point, e.g. at $X = 0$, the centre of the slot.

$$V_z(0,t)/V_c = \sum_{n=0}^{\infty} \frac{\mathrm{e}^{-(2n+1)^2(t/\tau)}}{(2n+1)^3}[(-1)^n - 3/(2n+1)\pi]. \tag{14}$$

In Fig. 2, the five first contributions to (14) ($n = 0, 1 \ldots 4$) are shown, together with their sum. It is clear that for $t/\tau \geqslant 0.6$, only the first contribution $n = 0$ is significant. When $t = t^* = 0.6\,\tau$, the dimensionless velocity amplitude is equal to 80% of the maximum value reached at $t/\tau \approx 0.3$. Consequently, for $t \geqslant t^*$, the sum (14) may be limited to the first term

$$V_z(0, t \geqslant t^*)/V_c = \mathrm{e}^{-t/\tau}(1 - 3/\pi) \tag{15}$$

and a logarithmic plot of V_z (0, $t \geqslant t^*$) yields the time constant τ and therefore the thermal diffusivity, knowing the slot width $2h$. Let us however recall that equation (15) was established for a quasi static process ($Pr \gg 1$), for a Boussinesq fluid, in the hypothesis of a pure conduction regime, which in turns implies a small Grashof number, or a small temperature step $T_f - T_i$.

STRAIGHT APPLICATION, POOR RESULTS

It should be interesting to determine the thermal diffusivity according to a process as simple as that given by equation (15). The experimental cell is the

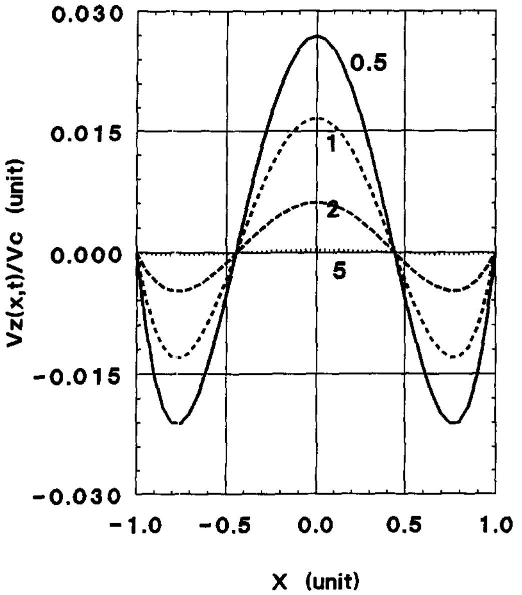

Fig. 1. Reduced velocity profile. Time relaxation to the rest state, for $t/\tau = 0.5, 1, 2, 5$.

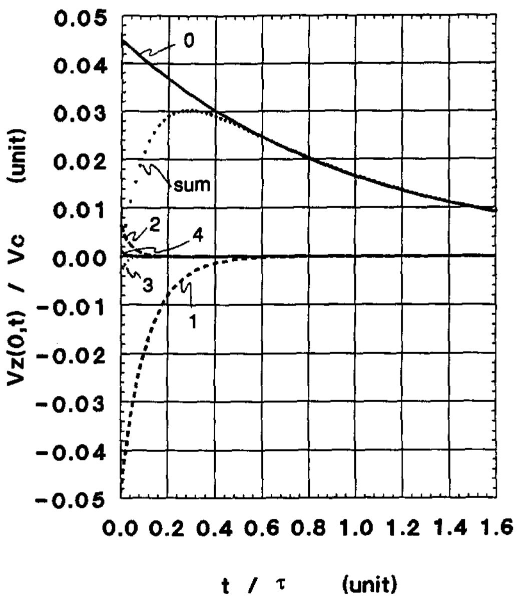

Fig. 2. Five first terms of equation (14), together with their sum.

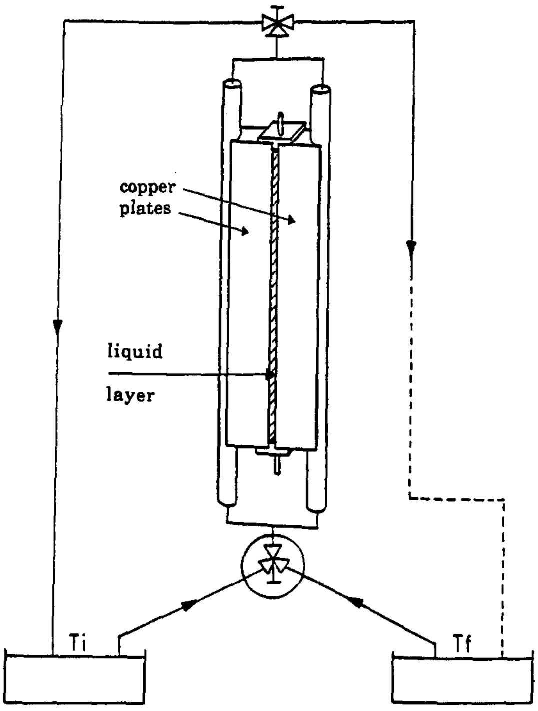

Fig. 3. Experimental cell.

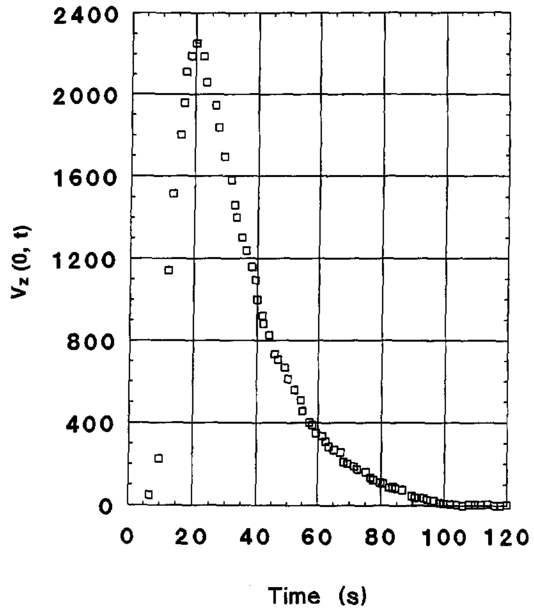

Fig. 4. Transient velocity amplitude at the centre of the slot, for water (T_{i}: 40°C; $T_{\mathrm{f}} = 20$°C). Run 1 of Table 1.

same as already used before [1], and sketched in Fig. 3. The cell width is 4.15 mm, so that the Grashof number remains small. The height is 28×4.15 mm, and therefore we hope that the assumption of an infinite aspect ratio is satisfactory (as demonstrated in [1]). The third dimension is 3.6×4.15 mm, and since the measurements are taken in the centre of the slot, we hope that lateral heat losses are negligible.

Experiments were performed first with water as working fluid and with the following constraint: $T_{\mathrm{i}} = 40$°C, $T_{\mathrm{f}} = 20$°C. In Fig. 4 the transient velocity amplitude in the centre of the slot recorded by LDV is plotted with real time scale in seconds.

Let us remark that the optical probe in LDV (a well known technique, not described here) has a typical diameter of 90 μm, and since it is located at $X = 0$ (or $x = 2.075$ mm, the centre of the slot) where the velocity is maximum (see Fig. 1), i.e. $\partial V_z/\partial x = 0$, the velocity gradient inside the optical probe is minimized. Of course this velocity gradient inside the probe is time dependent and is the greatest when the velocity reaches his maximum value at time $t = 20$ s in Fig. 4. The error on V_z can than be estimated: ± 100 μm s^{-1} or 5%

At $t < 0$, the temperature of the two copper lateral walls was fixed at 40°C. At the initial time $t = 0$, the temperature of these two walls is changed to 20°C. A few seconds are needed to push away the thermostating fluid at 40°C (therefore time $t = 0$ is not well defined experimentally); then the velocity amplitude grows rapidly and raises up to its maximum reached after 20 s. Finally, an "exponential" (?) decrease takes place and the rest state is practically recovered after 100 s. From Fig. 4, the natural logarithm of V_z is plotted vs time, for $t \geqslant t^* \approx 27$ s such that $V_z < 0.8\ V_{\mathrm{max}}$. The relationship looks fairly linear (see Fig. 5, where t^* is taken as zero time). Owing to the number of experimental points the error of 5% on each point should not affect too much the slope. According to equation (15), the characteristictime $\tau = 19.62$ s is deduced, and from (12), using $2h = 0.415$ cm, we find $\kappa = 8.9 \times 10^{-4}$ cm^2 s^{-1}. This is a disastrous result, since it is known for water that the value 14.42×10^{-4} cm^2 s^{-1} should be obtained. The experimental value is 35% too low, in comparison with the data reported in the literature. However the

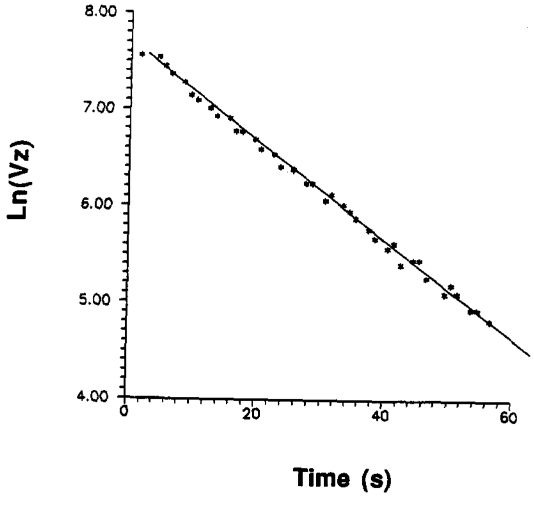

Fig. 5. Logarithmic plot of the velocity amplitude used for the determination of the relaxation time τ (water; $T_{\mathrm{i}} = 40$°C; $T_{\mathrm{f}} = 20$°C).

Table 1. Maximum velocity amplitude and determination of κ, for water ($T_i = 40°C$; $T_f = 20°C$)

Run	$V_{z\max}$ [μm s^{-1}]	$\kappa \times 10^4$ [cm^2 s^{-1}]
1	2258	8.90
2	2283	9.35
3	2324	9.03
4	2338	8.73
5	2258	9.07
6	2244	9.41
7	2258	9.52
8	2244	9.35
9	2270	9.28
		mean: 9.18
		true: 14.42

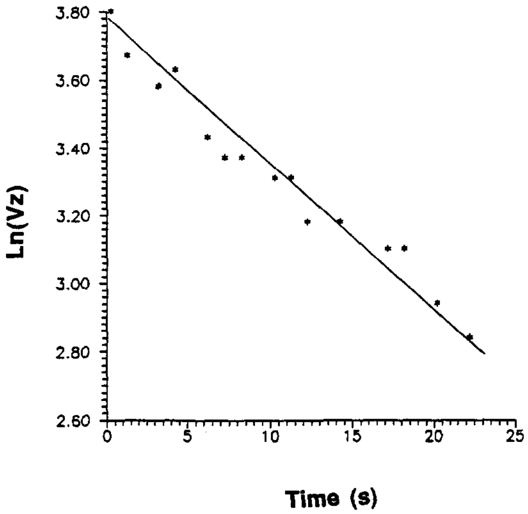

Fig. 6. Logarithmic plot of the velocity amplitude (isopropanol; $T_i = 21°C$; $T_f = 20°C$).

experiment is reproducible and Table 1 give the results of supplementary runs. Both maximum velocity and thermal diffusivity determination are reproducible, with a standard deviation of 2.8% for nine runs and a mean value of κ for water given by $\kappa = 9.18 \times 10^{-4}$ cm^2 s^{-1}, instead of 14.42×10^{-4} cm^2 s^{-1}. A possible explanation for this discrepancy could be in the fact that the Prandtl number for water is ≈ 7, a value that could be too small to support the assumption $Pr \gg 1$. Therefore the experiments were repeated with isopropanol as working fluid for which $Pr \approx 41$, and once again were reproducible (five runs) with the value of κ given in Table 2, corresponding to a mean value of 3.81×10^{-4} cm^2 s^{-1}, i.e. once again a too small value ($\approx 50\%$) compared with the data in the literature ($\kappa = 7.48 \times 10^{-4}$ cm^2 s^{-1}). Let us mention that the experiment on isopropanol was done for $\Delta T = 15°C$ ($T_i = 35°C$; $T_f = 20°C$). Thus the discrepancy is not due this time to the Prandtl number value.

NOT SO BAD, AT ZERO CONSTRAINT

The interpretation of the observed too low values of κ, could be found in the too high temperature differences applied ($\Delta T = 15°C$). Therefore the experiments were repeated for isopropanol for constraints ΔT down to 1°C. Lower constraints are forbidden due to the detection level of the LDV method. For each temperature difference ΔT, several runs are performed (typically five runs for large ΔT, up to 15 runs for $\Delta T = 1°C$ since this is the most difficult case owing the small values of V_z to be recorded). Figure 6 shows the "worst" case, an example among the 15 runs for $\Delta T = 1°C$. Table 3 gives finally the mean value of κ for different constraints ΔT and they are plotted in Fig. 7. With decreasing ΔT, κ converges to a value rather close to the correct one. By appropriate least squares fittings (3rd order polynomial) and extrapolation to $\Delta T = 0$, the computed data is 6.96×10^{-4} cm^2 s^{-1} instead of 7.48×10^{-4} cm^2 s^{-1} from literature. The whole set of experiments was repeated for ethanol with the following result, extrapolated at $\Delta T = 0$: $\kappa = 8.49 \times 10^{-4}$ cm^2 s^{-1} instead of $\kappa = 8.87 \times 10^{-4}$ cm^2 s^{-1}. Thus the extrapolation to $\Delta T = 0$ gives reasonable results. Nevertheless, the data obtained are lower (respectively 7 and 4%) than the values from literature. Remembering the dispersion of the process O(2%), some final improvement should be found.

IMPROVED METHOD

We believe that the remaining discrepancy lies in non-Boussinesq effect, not taken into account in equation (15). Let us take the viscosity temperature dependent. The effect is of order of 30% for isopropyl alcohol between 20 and 30°C. Of course, the consequence

Table 2. Determination of κ for isopropanol ($T_i = 35°C$; $T_f = 20°C$)

Run	$\kappa \times 10^4$ [cm^2 s^{-1}]
1	3.94
2	3.79
3	3.80
4	3.82
5	3.69
	mean: 3.81
	true: 7.48

Table 3. Variation of the deduced value of κ for different thermal constraints (case of isopropanol)

ΔT [°C]	$\kappa \times 10^4$ [cm^2 s^{-1}]
15	3.81
10	4.44
3.5	5.66
2	6.14
1.5	6.33
1	6.52
extrapolation to 0	6.96
	true: 7.48

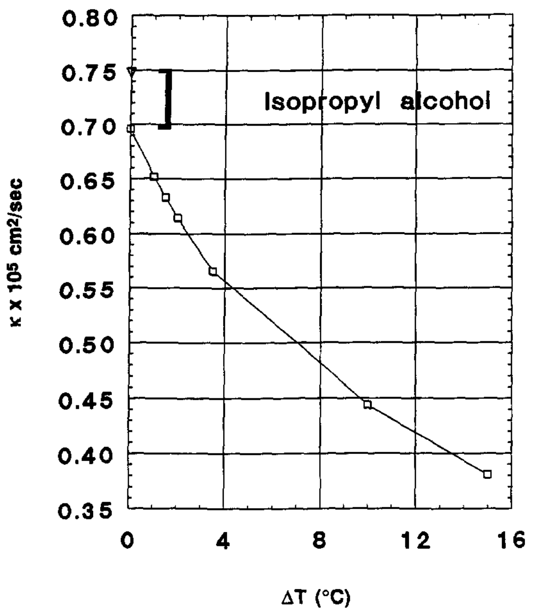

Fig. 7. Variation of the deduced value of κ for different thermal constraints and extrapolation to $\Delta T = 0$. ∇: value from the literature. The error bar is shown.

of viscosity variation has to be weak at ΔT O(1°C), but unfortunately the κ value is *extrapolated* and the final value is also determined by data recorded at high thermal constraints where viscosity effects can act significantly. Let us describe the temperature decrease of the dynamic viscosity by the law:

$$\mu = \mu_f/[1+\mu'(T-T_f)] \qquad (16)$$

where μ_f is the reference value at temperature T_f, and μ' is a constant depending of fluid. Therefore equation (8) is modified as follows:

$$\partial^2/\partial x^2[\mu(T)\,\partial V_z/\partial x] = -\alpha g \rho_f\,\partial T/\partial x. \qquad (17)$$

Dealing with the one-term ($n = 0$) approximation, from equation (3a), the temperature law is:

$$T-T_f = -\frac{4}{\pi}\Delta T\cos(\pi x/2h)\,\mathrm{e}^{-t/\tau}. \qquad (18)$$

By direct integration of equation (17), using (18), and after determination of the integration constants by the conditions (9) and (10), the velocity amplitude estimated in the centre position of the cell, ($x = 0$), becomes:

$$\frac{V_z(0,t>t^*)}{V_c} = \mathrm{e}^{-t/\tau} - \frac{\left(\frac{3}{\pi}\right)\mathrm{e}^{-t/\tau}+\delta A\,\mathrm{e}^{-2t/\tau}-\delta^2 B\,\mathrm{e}^{-3t/\tau}}{1+\delta C\,\mathrm{e}^{-t/\tau}} \qquad (19)$$

where V_c has the same value as before

$$\delta = -\mu'\Delta T = -\mu'(T_f-T_i)$$

$$A = \left(\frac{1}{3}-\frac{32}{\pi^2}+\frac{64}{\pi^3}\right)\frac{3}{2\pi} \approx -0.4034$$

$$B = \left(1+\frac{2}{\pi}-\frac{16}{\pi^2}\right)\frac{24}{\pi^3} \approx 0.0120$$

$$C = \frac{24}{\pi^2}\left(\frac{8}{\pi^2}-1\right) \approx -0.4606.$$

Expanding $1/(1+\delta C\,\mathrm{e}^{-t/\tau})$ in series, we get

$$\begin{aligned}\frac{V_z(0,t>t^*)}{V_c} \approx{}& \left(1-\frac{3}{\pi}\right)\mathrm{e}^{-t/\tau}-\left(-\frac{3}{\pi}\delta C+\delta A\right)\mathrm{e}^{-2t/\tau}\\ &-\left(\frac{3}{\pi}\delta^2C^2-\delta^2AC-\delta^2B\right)\mathrm{e}^{-3t/\tau}\\ &-\left(-\frac{3}{\pi}\delta^3C^3+\delta^3AC^2+\delta^3BC\right)\mathrm{e}^{-4t/\tau}+\cdots \qquad (20)\end{aligned}$$

or

$$\begin{aligned}\frac{V_z(0,t>t^*)}{V_c} \approx{}& 0.045\,\mathrm{e}^{-t/\tau}-\delta\times 0.0364\,\mathrm{e}^{-2t/\tau}-\delta^2\\ &\times 0.0048\,\mathrm{e}^{-3t/\tau}-\delta^3\times 0.0022\,\mathrm{e}^{-4t/\tau}+\cdots \qquad (21)\end{aligned}$$

Estimating the different terms at time $t/\tau = 0.63$ and for $\delta = 0.5$ (a reasonable value), we have

$$\begin{aligned}\frac{V_z(0;0.63)}{V_c} \approx{}& 2.40\times 10^{-2}-0.516\times 10^{-2}\\ &-0.018\times 10^{-2}\\ &-0.002\times 10^{-2} \qquad (22)\end{aligned}$$

and we see that the first correction in δ is not negligible since it represent more than 20% of the first term. However the term in δ^3 totally negligible. Owing to the values of A, B and C, we will take the following law for the determination of τ.

$$\begin{aligned}\frac{V_z(0,t>t^*)}{V_c} \approx{}& 0.045\,\mathrm{e}^{-t/\tau}\\ &-\delta\times 0.0364\,\mathrm{e}^{-2t/\tau}-\delta^2\times 0.0048\,\mathrm{e}^{-3t/\tau}. \qquad (23)\end{aligned}$$

Due to the form of equation (23), a direct estimation of τ by least squares fitting of the logarithm of V_z is no longer possible and an iteration process is required. Let us rewrite equation (23) in the form:

$$\frac{V_z(0,t>t^*)}{V_c} \approx 0.045\,\mathrm{e}^{-t/\tau}-F_1(\tau)-F_2(\tau). \qquad (24)$$

At the iteration 0, F_1 and F_2 are neglected and a first approximation of the relaxation time τ_0 is estimated from

$$\ln\frac{V_z(0,t>t^*)}{V_c\times 0.045} = -\frac{t}{\tau_0}. \qquad (25)$$

For iteration 1, $F_1(\tau_0)$ is taken into account in equation (24) and the next approximation of the relaxation time τ_1 is estimated from

$$\ln\left[\left(\frac{V_z}{V_c}+F_1(\tau_0)\right)/0.045\right] = -\frac{t}{\tau_1}. \qquad (26)$$

For the iteration 2, by the same way, one gets

$$\ln\left[\left(\frac{V_z}{V_c}+F_1(\tau_1)+F_2(\tau_1)\right)/0.045\right] = -\frac{t}{\tau_2}. \qquad (27)$$

From the successive iterations τ_0, τ_1, τ_2 we find the successive iterations for κ. They are listed in Table 4, both for isopropyl alcohol and ethyl alcohol together with the extrapolated value at $\Delta T = 0$. It is clear that the effect of the viscosity temperature dependence acts strongly at large ΔT. The convergence of the iterative process is clearly seen for the two sets of experiments. The final result is very fair. From Table 4, the measured value of κ for isopropyl alcohol (obtained at the second iteration and extrapolated to $\Delta T = 0$) is equal to 7.54×10^{-4} cm^2 s^{-1}. The value of the literature is 7.48×10^{-4} cm^2 s^{-1}, i.e. a difference of 0.8%. For ethyl alcohol, the final value of κ, obtained in the same conditions is 8.80×10^{-4} cm^2 s^{-1} instead of 8.87×10^{-4} cm^2 s^{-1}, i.e. also 0.8%. Finally, taking into account the viscosity temperature dependence, the κ values, measured by LDV in the transient convective regime, fit the reference data from literature within an error which corresponds to the standard deviation of the method O(1%). There is of course no more difficulty in using equation (23) than equation (15). The only thing needed is the value of δ, or of μ'. There is thus a need to know the viscosity at least at T_i and T_f, from literature or own measurements, and this is rather easy. Finally, instead of finding μ' or δ from a separate fitting of the experimental values of the viscosity to equation (16), one could try to find *simultaneously* the two parameters τ and δ by fitting the experimental points (as recorded e.g. on Fig. 4) directly to equation (23) (see [8]). This procedure failed to give precise results: do not expect too much from the method proposed in the present paper, i.e. the simultaneous determination of the thermal diffusivity κ, of the temperature dependence of the viscosity μ', and why not the thermal expansion coefficient α, hidden in V_c!

Finally from Fig. 2, the maximum velocity amplitude reached at $t/\tau \approx 0.3$ should be proportional to V_c, or to ΔT, at least as long as the conductive regime is assumed. This is indeed what is observed in the present experiments (see Fig 8).

CONCLUSION

By using an *ad hoc* equation for the transient velocity amplitude, assuming

(i) the quasi-static approximation (or equivalently $Pr \gg 1$),

(ii) the conduction regime approximation (small Grashof numbers) and

(iii) non-Boussinesq effects (viscosity is temperature dependent),

accurate values of the thermal diffusivity can be obtained (say within 1% error).

Therefore, the final conclusion, which could also be the title of the present paper, is: why not use the

Table 4. Successive iterations for κ, when taking into account the temperature dependence of the viscosity: (a) isopropanol and (b) ethanol

(a)

ΔT	$\kappa^{(0)} \times 10^4$ [cm^2 s^{-1}]	$\kappa^{(1)} \times 10^4$ [cm^2 s^{-1}]	$\kappa^{(2)} \times 10^4$ [cm^2 s^{-1}]
15	3.81	4.55	4.67
10	4.44	5.13	5.23
3.5	5.66	6.03	6.04
2	6.14	6.50	6.52
1.5	6.33	6.74	6.80
1	6.52	6.93	6.97
	extrapolation to $\Delta T = 0$		7.54
			true: 7.48

(b)

ΔT	$\kappa^{(0)} \times 10^4$ [cm^2 s^{-1}]	$\kappa^{(1)} \times 10^4$ [cm^2 s^{-1}]	$\kappa^{(2)} \times 10^4$ [cm^2 s^{-1}]
10	4.67	5.03	5.06
7	4.79	5.11	5.14
5	4.91	5.13	5.14
4	5.05	5.20	5.21
3	5.26	5.45	5.46
2	6.11	6.26	6.26
1	7.06	7.27	7.27
	extrapolation to $\Delta T = 0$		8.80
			true: 8.87

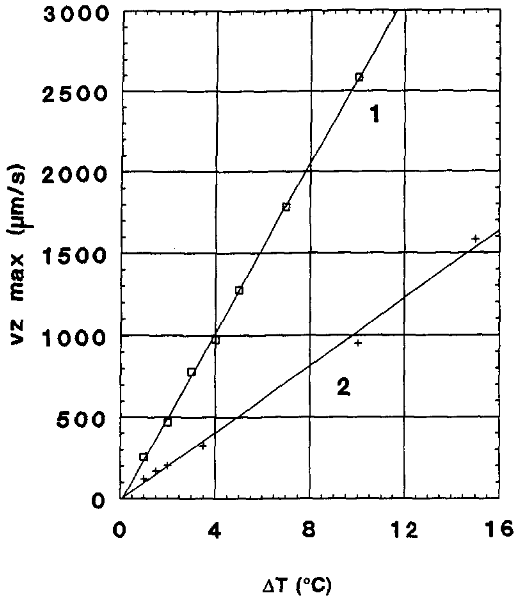

Fig. 8. Variation of the maximum recorded transient velocity with ΔT. Curve 1 : ethanol. Curve 2 : isopropanol.

non-intrusive LDV method to determine the thermal diffusivity of liquids?

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